Extended Open Inflation

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We present a new type of one-field models for open inflation utilizing a nonminimally coupled scalar field with polynomial potentials in which the Coleman-de Luccia instanton does exist and slow-roll inflation after the bubble nucleation is realized.

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The possibility of the creation of an open universe within the context of the inflationary scenario [1–4] is intriguing in that it enriches predictions of inflationary cosmology and that the horizon problem and the flatness problem can be solved separately within the inflationary paradigm. The basic idea is that a symmetric bubble nucleates in de Sitter space [5,1] and its interior undergoes a second stage of slow-roll inflation adjusting Ω_0 in the rage $0 < \Omega_0 < 1$.

As noted by Linde, however, it is very difficult to satisfy both $|V''|/H^2 > 1$ (the condition for the existence of Coleman-de Luccia instantons) and $|V''|/H^2 \ll 1$ (for slow-roll inflation after the bubble nucleation) [6]. In fact, we did not have any consistent models until Linde proposed a simple one-field model [6](see also [7]). The purpose of the present paper is to propose some version of open inflation satisfying both requirements using a nonminimally coupled scalar field with polynomial potentials. We shall name our model "extended" open inflation in the sense of [8]. The possibility of open inflation with a nonminimally coupled scalar field has been suggested in [9]. However, as far as we know, the concrete realization has not appeared in the literature* (we note that variants of the Hawking-Turok instanton for creation of an open universe [10] with a nonminimally coupled scalar field were considered in [11]).

Linde's potential consists of two parts:

$$V(\phi) = \frac{1}{2}m^2\phi^2 + \frac{1}{2}m^2\phi^2 \frac{1}{\beta^2/\alpha^2 + (\phi - v)^2/\alpha^2} \equiv V_1(\phi) + V_2(\phi)\frac{1}{f(\phi)^2}, \tag{1}$$

where the first term, $V_1(\phi)$, controls inflation after quantum tunneling, while the second term, $V_2(\phi)/f(\phi)^2$, controls the bubble nucleation. The essential role of $f(\phi)$ is to create a dip in the potential $V(\phi)$ around the minimum of $f(\phi)$. Somewhat *ad hoc* feature of this model is the appearance of the second term in which a polynomial potential $V_2(\phi)$ is divided by another polynomial function $f(\phi)^2$. Thus if the potential of the form $f(\phi)^2 V(\phi)$ appears at the very introduction of the action and the potential $V(\phi)$ appears as an *effective* potential, then we may avoid the unnaturalness to some extent. These considerations suggest a nonminimal coupling of the scalar field to gravity in the following form:

$$S = \int d^4x\sqrt{-g}\left[\frac{1}{2\kappa^2}f(\phi)R - \frac{1}{2}g^{ab}\partial_a\phi\partial_b\phi - f(\phi)^2 V_1(\phi) - V_2(\phi)\right], \tag{2}$$

where $\kappa^2 = 8\pi G_*$ and G_* is the bare gravitational constant. Indeed after the conformal transformation of the form

$$g_{Eab} = f(\phi) g_{ab}, \tag{3}$$

the action Eq.(2) reduces that of the scalar field minimally coupled to the Einstein gravity:

$$S = \int d^4x\sqrt{-g_E}\left[\frac{1}{2\kappa^2}R_E - \frac{1}{2}F(\phi)^2(\nabla_E\phi)^2 - V_E(\phi)\right], \tag{4}$$

where

$$F(\phi)^2 = \frac{1}{f(\phi)^2}\left(f(\phi) + \frac{3f(\phi)_{,\phi}{}^2}{2\kappa^2}\right), \tag{5}$$

$$V_E(\phi) = V_1(\phi) + V_2(\phi)\frac{1}{f(\phi)^2}. \tag{6}$$

Here the subscript represents the functional derivative with respect to ϕ. Apart from the non-canonical kinetic term, the potential $V(\phi)$ in Eq.(4) has essentially the same feature as Linde's model given in Eq.(1).

The appearance of the potential terms of different coupling to ϕ may seem rather *ad hoc*. We speculate on a possible origin of these potentials. One possibility is a massive field with a potential of the form $V_1(\phi)$ in the string frame [12]. Hence we may regard the Lagrangian Eq.(2) without $V_2(\phi)$ as a starting point. Another potential $V_2(\phi)$ may come from a nonperturbative effect associated with a dynamical symmetry breaking.

An $O(4)$-symmetric instanton with the metric (in the original frame g_{ab})

$$ds^2 = d\tau^2 + a(\tau)^2(d\psi^2 + \sin^2\psi d\Omega_2^2), \tag{7}$$

*In fact, in [6], Linde noted "... We also tried to use scalar fields nonminimally coupled to gravity. All these attempts so far did not lead to a successful one-field open universe scenario".

which describes the creation of an open universe should satisfy the following equations of motion

$$\frac{a''}{a} = -\frac{\kappa^2}{3f}\left(\phi'^2 + V\right) - \frac{f_{,\phi}}{2f}\frac{a'}{a}\phi' - \frac{f_{,\phi\phi}}{2f}\phi'^2 - \frac{f_{,\phi}}{2f}\phi'', \quad (8)$$

$$\phi'' = -3\frac{a'}{a}\phi' + V_{,\phi} + \frac{3f_{,\phi}}{\kappa^2}\left(\frac{a''}{a} + \left(\frac{a'}{a}\right)^2 - \frac{1}{a^2}\right), \quad (9)$$

where primes denote derivatives with respect to τ. A regular instanton should satisfy the boundary conditions such that $\phi' = 0$ and $a' = \pm 1$ at $a = 0$. After a proper analytic continuation to the Lorentzian region, the configuration describes an open universe residing inside an expanding bubble.

As a concrete example, let us consider the following functional forms[†]

$$f(\phi) = 1 + \xi\kappa^2\phi^2, \quad (10)$$

$$V_1(\phi) = V_2(\phi) = \frac{1}{2}m^2(\phi - v)^2. \quad (11)$$

Clearly, this is not a unique choice. In particular, the mass scale of $V_1(\phi)$ needs not to coincide with that of $V_2(\phi)$. Moreover, a different functional form for V_2, for example $V_2(\phi) = \lambda(\phi^2 - v^2)^2$, may be possible. However, $V_2(\phi)$ should have the same minimum as $V_1(\phi)$. $f(\phi)$ should be convex and positive, and its curvature around the minimum should be larger than that of V_1 and V_2. For definiteness, we choose $\xi = 1$ and $v = 10 \times m^*_{pl}$ with $m^{*2}_{pl} \equiv G^{-1}_*$, and then $m = 8 \times 10^{-7} m^*_{pl}$ for the normalization of the density perturbations. Note that ξ needs not always to be large. The shape of the effective potential $V_E(\phi)$ is shown in Fig. 1.[‡] We note that this choice of the parameters is not a unique one. For example, another possible choice is $\xi = 100, v = 1.5 \times m^*_{pl}$ and $m = 4 \times 10^{-6} m^*_{pl}$. In Fig. 2, we show the Coleman-de Luccia instanton of our model.

[†]We note that we could in fact find a viable model (that is, both Coleman-de Luccia instanton and slow-roll inflation after the tunneling with reasonable e-fold is realized) with $V_1 = 0$ and up to quartic terms for V_2. But a considerable fine tuning for the parameters is required, and the results are not so illuminating. We hesitate to show them here.

[‡]We should draw the figure using the canonically normalized scalar field Φ defined by $\Phi = \int d\phi F(\phi)$. However, the shape does not change qualitatively.

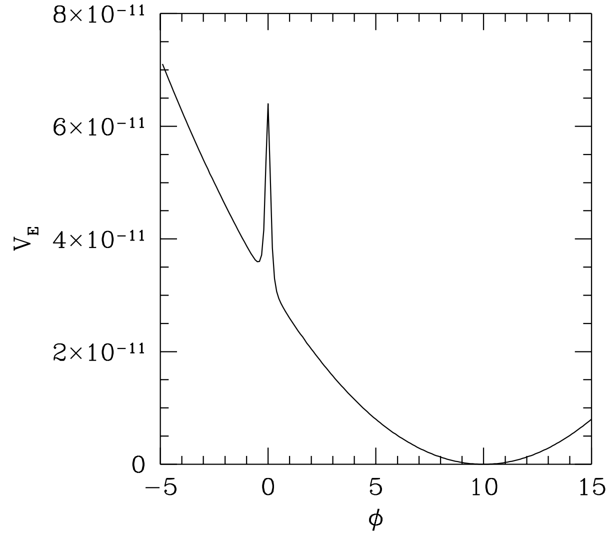

Fig.1

FIG. 1. The effective potential $V_E(\phi)$. All values are normalized by m^*_{pl}.

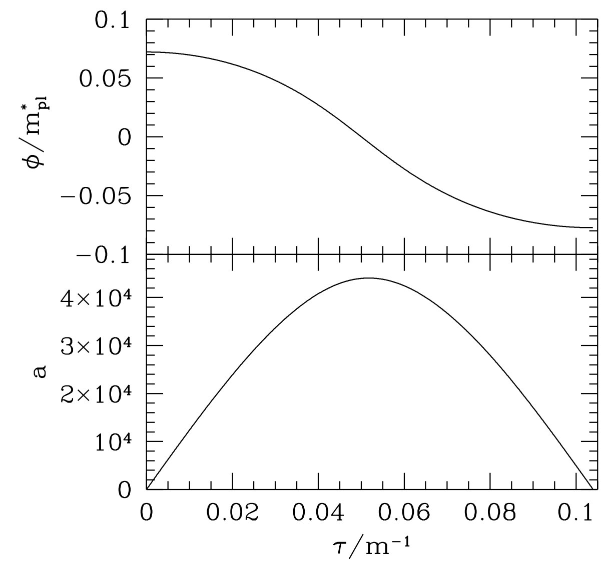

Fig.2

FIG. 2. Coleman-de Luccia instanton in our model. The upper panel shows the scalar field; the lower panel shows the scale factor.

After the tunneling, the universe undergoes the inflationary stage. The scale factor and the scalar field obey the following equations after analytically continued to the Lorentzian regime

$$\frac{\ddot{a}}{a} = -\frac{\kappa^2}{3f}\left(\dot{\phi}^2 - V\right) - \frac{f_{,\phi}}{2f}\frac{\dot{a}}{a}\dot{\phi} - \frac{f_{,\phi\phi}}{2f}\dot{\phi}^2 - \frac{f_{,\phi}}{2f}\ddot{\phi}, \quad (12)$$

$$\ddot{\phi} = -3\frac{\dot{a}}{a}\dot{\phi} - V_{,\phi} + \frac{3f_{,\phi}}{\kappa^2}\left(\frac{\ddot{a}}{a} + \left(\frac{\dot{a}}{a}\right)^2 - \frac{1}{a^2}\right), \qquad (13)$$

where dots denote derivatives with respect to the cosmic time. The boundary conditions are $\dot{\phi} = 0$ and $\dot{a} = 0$ at $a = 0$. Then, the evolution of the scalar field and the scale factor are depicted in Fig. 3.

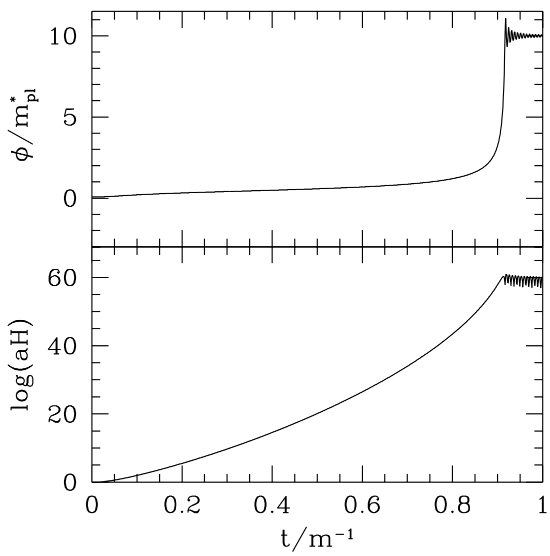

Fig.3

FIG. 3. Inflation after the tunneling. The upper panel shows the evolution of the scalar field. The lower panel shows the number of e-folds of the decrease of the comoving Hubble scale.

We calculate the function $\Delta = H^2/(5\pi\sqrt{1+6\xi}|\dot{\phi}|)$ which would correspond to density perturbations measured on a comoving hypersurface in a flat matter-dominated universe [13]. Similar to Linde's model, the amplitude of Δ has a maximum at small $\log(aH)$ as shown in Fig. 4. In our model, the maximum is at $\log(aH) \simeq 20$. It should be noted that the true Planck scale m_{pl} after inflation ($\phi = v$) is given by $m_{pl} = m^*_{pl}\sqrt{1+\xi\kappa^2 v^2}(> m^*_{pl})$. Therefore ϕ during inflation can be smaller than m_{pl} although the effective Planck scale at the beginning of inflation $m^{eff}_{pl} \equiv m^*_{pl}\sqrt{1+\xi\kappa^2\phi^2}$ is not so much different from m^*_{pl}. For example, for the parameters given above, $m_{pl} \simeq 50 \times m^*_{pl}$ and thus $v \simeq 0.2 \times m_{pl}$.

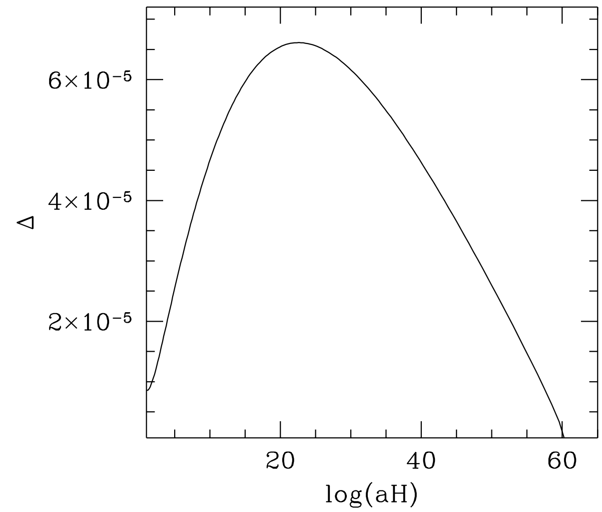

Fig.4

FIG. 4. Density perturbations Δ produced inside the bubble N e-folds after the creation of an open universe.

To conclude, allowing the nonminimal coupling of the inflaton to gravity greatly expands the range of viable models for open inflation.

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